<u>**Using the limit definition of the definite integral, and right endpoints,**</u> find $\int_{-2}^{3} (x^2 + 4x - 4)\, dx$.

SCORE: _____ / 15 PTS

<u>**NOTE: Solutions using any other method will earn 0 points.**</u>

$$\Delta x = \frac{3 - -2}{n} = \frac{5}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\Big(\underbrace{-2 + \frac{5i}{n}}_{1}\Big)\underbrace{\frac{5}{n}}_{1}$$

3

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\Big(\Big(-2 + \frac{5i}{n}\Big)^2 + 4\Big(-2 + \frac{5i}{n}\Big) - 4}\Big)\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \Big(\cancel{4} - \cancel{\frac{20i}{n}} + \frac{25i^2}{n^2} - 8 + \cancel{\frac{20i}{n}} - \cancel{4}\Big)\frac{5}{n}$$

$$= \lim_{n\to\infty} \frac{5}{n} \sum_{i=1}^{n} \underbrace{\Big(\frac{25i^2}{n^2} - 8\Big)}_{3}$$

$$= \lim_{n\to\infty} \frac{5}{n}\Big(\frac{25}{n^2}\sum_{i=1}^{n} i^2 - 8n\Big)$$

$$= \lim_{n\to\infty} \frac{5}{\cancel{n}}\Big(\underbrace{\frac{25}{n^2}\frac{n(n+1)(2n+1)}{6}}_{2} - \underbrace{8\cancel{n}}_{2}\Big)$$

$$= \underbrace{5\Big(\frac{25 \cdot \cancel{2}}{\cancel{6}_3} - 8\Big)}_{1}$$

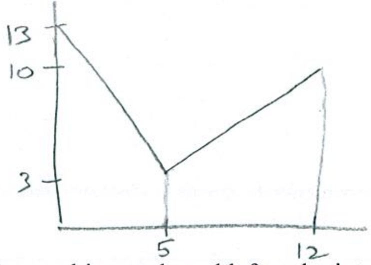
$$= \underbrace{\frac{5}{3}}_{1}$$

1 IF YOU WROTE $\lim_{n\to\infty} \sum_{i=1}^{n}$ ON EVERY LINE BEFORE THIS

A person's velocity (in meters per ~~minute~~ SECOND) at time t (in ~~minutes~~ SECONDS) is given by $v(t)=\begin{cases}13-2t, & 0\le t\le 5\\ t-2, & 5\le t\le 12\end{cases}$.

SCORE: _____ / 5 PTS

[a] Find the exact distance the person travelled from time $t=0$ seconds to $t=12$ seconds.



$\frac{1}{2}(13+3)(5-0)+\frac{1}{2}(3+10)(12-5)$

$=\underbrace{\frac{1}{2}(13+3)(5)}_{1}+\underbrace{\frac{1}{2}(13)(7)}_{1}$

$=40+\frac{91}{2}=\underbrace{\frac{171}{2}\text{ OR }85\frac{1}{2}}_{\frac{1}{2}}$ METERS

[b] Estimate the distance the person travelled from time $t=0$ seconds to $t=12$ seconds using three subintervals and left endpoints.

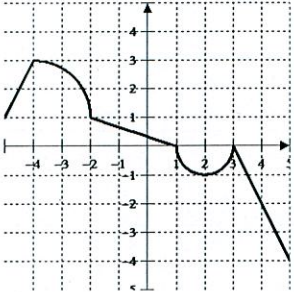
$\Delta t=\frac{12-0}{3}=4$

0 4 8 12

$v(0)\Delta t+v(4)\Delta t+v(8)\Delta t=\underbrace{13(4)}_{1}+\underbrace{5(4)}_{\frac{1}{2}}+\underbrace{6(4)}_{\frac{1}{2}}=\underbrace{96}_{\frac{1}{2}}$ METERS

The graph of function f is shown on the right. SCORE: _____ / 5 PTS

The graph consists of 3 diagonal lines, alternating with two arcs of circles.



[a] Evaluate $\int_{-5}^{5} f(x)\,dx$.

$\frac{1}{2}$ POINT EACH

$$= \int_{-5}^{1} f(x)\,dx + \int_{1}^{5} f(x)\,dx$$

$$= \left[\tfrac{1}{2}(1+3)(1) + \left(\tfrac{1}{4}\pi(2)^2 + 1(2)\right) + \tfrac{1}{2}(1)(3)\right)$$

$$- \left[\tfrac{1}{2}\pi(1)^2 + \tfrac{1}{2}(4)(2)\right] = 2 + \pi + 2 + \tfrac{3}{2} - \left[\tfrac{1}{2}\pi + 4\right] = \frac{\pi}{2} + \frac{3}{2}$$

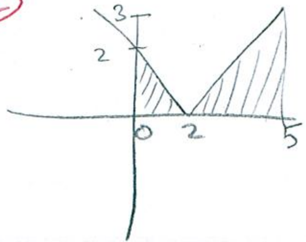
[b] Evaluate $\int_{3}^{-2} f(x)\,dx$.

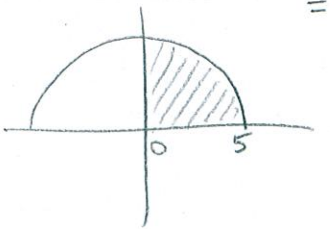
$$= -\int_{-2}^{3} f(x)\,dx = -\left[\int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx\right] = -\left[\tfrac{3}{2} - \tfrac{1}{2}\pi\right] = \frac{\pi}{2} - \frac{3}{2}$$

Evaluate $\int_0^5 (|x-2| - 2\sqrt{25-x^2})\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

$= \int_0^5 |x-2|\,dx - 2\int_0^5 \sqrt{25-x^2}\,dx = \frac{1}{2}(2)(2) + \frac{1}{2}(3)(3) - 2\left(\frac{1}{4}\pi(5)^2\right)$

2





$= \frac{13}{2} - \frac{25\pi}{2}$

$1\frac{1}{2}$ $1\frac{1}{2}$